Phase-locking and frustration in an array of nonlinear spin-torque nano-oscillators

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Abstract

We demonstrate that high-frequency cooperative dynamics of an array of coupled nonlinear spin-torque nano-oscillators (STNO) can be controlled by introduction of an additional external phase shift β_c between the microwave current, which couples STNOs, and the total array microwave voltage. When this external phase shift β_c compensates the intrinsic phase shift β_0, caused by the STNO nonlinearity, a *phase-locking* regime with increased output power and vanishing inhomogeneous linewidth broadening is achieved. In the opposite case, when external and intrinsic phase shifts are added, the STNO array demonstrates a *frustration* regime with low output power and wide and noisy frequency spectrum.

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The major problems in the practical realization of spin-torque nano-oscillators (STNO) [1–4] are their low output power and large generation linewidth. Both these problems can be solved by using the phenomenon of mutual phase-locking in an array of coupled STNOs [5–9].

In contrast with the majority of conventional oscillators, STNOs are strongly nonlinear: both the generated frequency $\omega(p)$ and the total damping $\Gamma(p)$ strongly depend on the generated power p (see [10] for details). It is convenient to characterize the STNO nonlinearity by a dimensionless parameter (see Eq. (33) in [10]):

$$\nu = \frac{d\omega(p)/dp}{d\Gamma(p)/dp}\,, \tag{1}$$

which for typical STNO parameters could have the magnitude as large as $|\nu| \sim 100$.

The nonlinearity significantly influences the interaction of STNOs with external signals and with each other. In particular, it substantially increases the frequency band Δ of phase-locking of STNO to an external signal (compared to the linear value Δ_0 of this band) [8, 11, 12] and it creates a large (about $\pm 90^\circ$) intrinsic phase shift β_0 [13–15] between the driving signal and the oscillations of STNO resistance (see Eqs. (52), (53) in [10]):

$$\Delta = \sqrt{1+\nu^2}\Delta_0\,, \qquad \beta_0 = -\arctan(\nu)\ . \tag{2}$$

The intrinsic phase shift β_0 does not change the conditions of phase-locking of an STNO to an *external* signal, the phase of which is independent of the phase of STNO, but may be of a critical importance for the process of *mutual* phase-locking in coupled STNO array, where the driving signal is created by all the STNOs in the array.

In this work we demonstrate that the large intrinsic phase shift β_0 can completely suppress mutual phase-locking in an STNO array. We also show that the disruptive influence of β_0 can be compensated by introduction of an additional external phase shift β_c of a proper sign between the total microwave current driving the array (and created by all the STNOs in the array) and the total microwave voltage. The introduction of this additional phase shift β_c does not affect the nonlinear enhancement of the phase-locking band (see first equation in (2)).

The mutual phase-locking of STNOs was studied numerically on the example of an array shown in Fig. 1. It consists of $N = 10$ nano-pillar STNOs electrically connected in series, biased by the dc current $I_0 = 1$ mA, and connected in parallel with an external RLC circuit.

The dynamics of each STNO is described by the Landau-Lifshits-Gilbert-Slonczewski equation [1] for the magnetization vector $\boldsymbol{M}_i$ of "free" layer of i-th STNO:

$$\begin{aligned}\dot{\boldsymbol{M}}_i &= \gamma[\boldsymbol{H}_i \times \boldsymbol{M}_i] + \frac{\alpha_{\mathrm{G}}}{M_0}[\boldsymbol{M}_i \times \dot{\boldsymbol{M}}_i] \\ &+\frac{\sigma_i I(t)}{M_0}[\boldsymbol{M}_i \times [\boldsymbol{M}_i \times \hat{\boldsymbol{p}}]] \ .\end{aligned} \tag{3}$$

Here $\gamma = 2\pi \cdot 2.8$ MHz/Oe, $\boldsymbol{H}_i = (H_0 - 4\pi M_{i,z})\hat{\boldsymbol{z}}$, $H_0 = 10$ kOe is the bias magnetic field directed perpendicularly to the "free" layer (along coordinate $\hat{\boldsymbol{z}}$), and term $-4\pi M_{i,z}$ is the dipolar field responsible for the nonlinearity of STNO. The second term in Eq. (3) is the Gilbert damping term, $\alpha_{\mathrm{G}} = 0.01$ and $4\pi M_0 = 8$ kOe are the Gilbert damping constant and saturation magnetization of the "free" layer, respectively.

The last term in Eq. (3) is the spin-transfer torque which describes the action of the electric current $I(t)$ on the i-th STNO. Here $\sigma_i = \varepsilon_i \gamma \hbar/(2eM_0V_0)$, ε_i is the dimensionless spin-polarization efficiency for i-th STNO, $\hbar = h/(2\pi)$ is the reduced Planck constant, e is the modulus of electron charge, $V_0 = 2.35 \times 10^4$ nm^3 is the volume of the "free" STNO layer (which corresponds to the circular nano-pillar of the radius 50 nm and thickness 3 nm), and $\hat{\boldsymbol{p}} = \sin\theta_p \, \hat{\boldsymbol{x}} + \cos\theta_p \, \hat{\boldsymbol{z}}$ is the direction of spin polarization of the current ($\theta_p = 30^\circ$ in our calculations).

Spin-polarization efficiencies ε_i were chosen to be different for different STNOs (1 % spread around the central value of $\varepsilon_0 = 0.4$), thus creating a spread of the free-running frequenies of uncoupled STNOs in the interval $\Delta f_0 \approx 100$ MHz around the central generation frequency of $f_0 \approx 12.16$ GHz (see Fig. 2a and discussion below).

The electric current $I(t)$ consists of the dc bias current I_0 and microwave current $\Delta I(t)$ (see Fig. 1) created by the magnetization precession of each STNO due to giant magnetoresistance (GMR) or tunneling magnetoresistance (TMR) effect. For the considered electrical scheme Fig. 1 $\Delta I = dQ/dt$, where Q is the charge on the capacitance C, which obeys the equation

$$L\ddot{Q} + \left[R + \sum_i R_i(t)\right]\dot{Q} + Q/C = -\sum_i R_i(t) I_0 \, , \tag{4}$$

where $R_i(t)$ is the resistance of the i-th STNO:

$$R_i(t) = R_0 + \Delta R_i(t) = R_0 \left[1 + \frac{\mu}{2}\left(1 - \cos\theta_i(t)\right)\right] \ . \tag{5}$$

Here $R_0 = 10$ Ohm is the electric resistance of STNO in parallel magnetic state, $\mu = 0.3$

is the magnetoresistance of the oscillators, and $\theta_i(t) = \arccos(\boldsymbol{M}_i(t) \cdot \hat{\boldsymbol{p}}/M_0)$ is the time-dependent angle between the magnetization directions in the "free" and "fixed" magnetic layers.

The microwave current $\Delta I(t)$, created by *all* the STNOs and acting on *each* of them, serves not only as the output signal from the STNO array, but, also, is the coupling signal that can drive the array of oscillators.

The role of the RLC circuit, placed in parallel with the STNO array, was to create an external phase shift $\beta_c = \arctan((f_c - f_0)/\Delta f_c)$ (where $f_c = 1/(2\pi\sqrt{LC})$, $\Delta f_c = (R + NR_0)/(4\pi L) = 1$ GHz, and f_0 is the generation frequency of the array) between the microwave current $\Delta I(t)$ flowing through the array and the generated microwave voltage $\mathcal{E}(t) = I_0 \sum_i \Delta R_i(t)$.

The results of the numerical modeling of the dynamics of the STNO array Fig. 1 are presented in Fig. 2, which demonstrates temporal profiles of the microwave current $\Delta I(t)$ (left column) and spectral density of power on the active load $R = 50$ Ohm (right column) for several different parameters of the RLC circuit.

The upper row (Fig. 2(a)) shows the case of uncoupled STNOs, when the feedback action of the common microwave current $\Delta I(t)$ on the STNO dynamics was neglected. The power spectrum in this case consists of $N = 10$ narrow peaks generated by each of STNOs. The incoherent sum of $N = 10$ independent oscillations with different frequencies leads to chaotic-like temporal profile of the current $\Delta I(t)$ (see Fig. 2(a), left column).

Fig. 2(b) shows the case of *coupled* STNO array when the resonance frequency of the RLC circuit was exactly equal to the central generation frequency $f_c = f_0 = 12.16$ GHz (the external phase shift was $\beta_c = 0$).

It is important to stress, that, if one ignores the intrinsic phase shift β_0, the generated current $\Delta I \sim 20$ μA is sufficient to have phase-locking in the frequency interval ~ 300 MHz [8], which is several times larger than the spreading of the free-running frequencies of our STNO array. Thus, naively, one may expect that the coupled STNO array will be completely phase-locked.

The numerical results Fig. 2(b), that implicitly take into account the phase shift β_0, prove otherwise. The temporal profile of the current $\Delta I(t)$ is still quasi-chaotic (left column), while its power spectrum is wide and noisy, and is shifted to the region of lower frequencuces (right column). Similar shift (only to the region of higher frequencies) was found in simulations

[16] performed for the case of *in-plane* magnetized STNO array (i.e., STNOs having opposite sign of the nonlinearity ν Eq. (1)).

The reason for this incoherent behavior of the array is the large intrinsic phase shift β_0, that for the parameters of our array can be evaluated using [10] as $\beta_0 \approx -\arctan(\nu) \approx -89.4^\circ$. Due to this large phase shift the oscillations in each STNO dynamically adjust their phases roughly 90° behind the phase of the current $\Delta I(t)$, and, since the driving current $\Delta I(t)$ itself is created by the STNOs, such a 90° phase delay destroys any coherence that can spontaneously arise in the array.

A very different picture is seen when one introduces an external *positive* phase shift $\beta_c = \arctan((f_c - f_0)/\Delta f_c) = 45^\circ$, that partly compensates the intrinsic phase shift β_0 to give the total phase shift of $\beta = \beta_0 + \beta_c \approx -45^\circ$. In that case (see Fig. 2(c)) the array enters a coherent phase-locked generation regime with narrow frequency spectrum and large output power.

To demonstrate that the observed qualitative change of the STNO array dynamics is caused solely by the phase mechanism, we performed simulations for the case when the external phase shift introduced by the RLC circuit is *negative* $\beta_c = -45^\circ$, thus making the total phase shift $\beta = \beta_0 + \beta_c \approx -135^\circ$ (see Fig. 2(d)). In this case the dynamics of the STNO array qualitatively differs from all the previously considered cases. The generation spectrum remains chaotic (as in Fig. 2(b)), but the amplitude of the current ΔI is drastically reduced. This reduction is not related to the increase of the RLC circuit impedance (which was exactly the same in the cases (c) and (d)). Our additional simulations with a different number N of oscillators indicate that the magnitude of the current *decreases* with the *increase* of the number of interacting oscillators and completely vanishes in the limit $N \to \infty$.

This counter-intuitive result can be understood by noting that for the large phase shifts $|\beta| > 90^\circ$ the interaction between the STNOs has a repulsive nature, i.e. oscillators dynamically adjust their phases to *minimize* the mutual coupling. Therefore, the observed drastic reduction of the coupling current ΔI in this *frustration* cooperative regime is a result of the *dynamical destructive interference* of oscillations of different STNOs.

Our simulation results can be qualitatively explained using the modified Kuramoto model of coupled auto-oscillators, which takes into account phase shift β. In the framework of this

model, each STNO is characterized by only its phase $\phi_i(t)$ (see [10] for details):

$$\dot{\phi}_i = \omega_i + \Lambda \sum_j \sin(\phi_j - \phi_i + \beta) \ . \tag{6}$$

Here $\omega_i = 2\pi f_i$ are the free-running oscillator frequencies, $\Lambda > 0$ describes the interaction strength between the oscillators, and $\beta = \beta_0 + \beta_c$ is the total phase shift.

In the limit of large coupling $\Lambda \to \infty$ the system (6) has the coherent stationary solution

$$\phi_i(t) = \omega t \,, \qquad \omega \simeq \omega_0 + N\Lambda \sin\beta \ . \tag{7}$$

Eq. (7) explains the above mentioned shift of averaged frequency ω in the coupled STNO array. Linear stability analysis shows that the coherent solution Eq. (7) is stable only for $\cos\beta > 0$, i.e. for $|\beta| < 90°$. For larger angles $|\beta| > 90°$ ($\cos\beta < 0$) the coherent dynamics of STNO array is impossible independently of the interaction strength Λ. In this case the incoherent (or *frustrated*) state with evenly distributed phases ϕ_i becomes stable. The total output power ($\propto |\sum_j \exp[i\phi_j(t)]|^2$), vanishes completely in this frustrated state.

In conclusion, we have shown that the cooperative dynamics in an array of coupled STNOs can be controlled by the introduction of an additional external phase shift β_c to the microwave coupling signal. By changing β_c, STNO array can be driven into one of three qualitatively different regimes: coherent phase-locked regime (see Fig. 2(c)) with large output power and narrow generation linewidth, intermediate chaotic regime (see Fig. 2(b)) with wide spectrum and relatively large output power, and the *frustration* regime (see Fig. 2(c)) with a vanishingly small output power. We believe that our results will be important for the design of practical oscillators based on the phase-locked arrays of STNOs.

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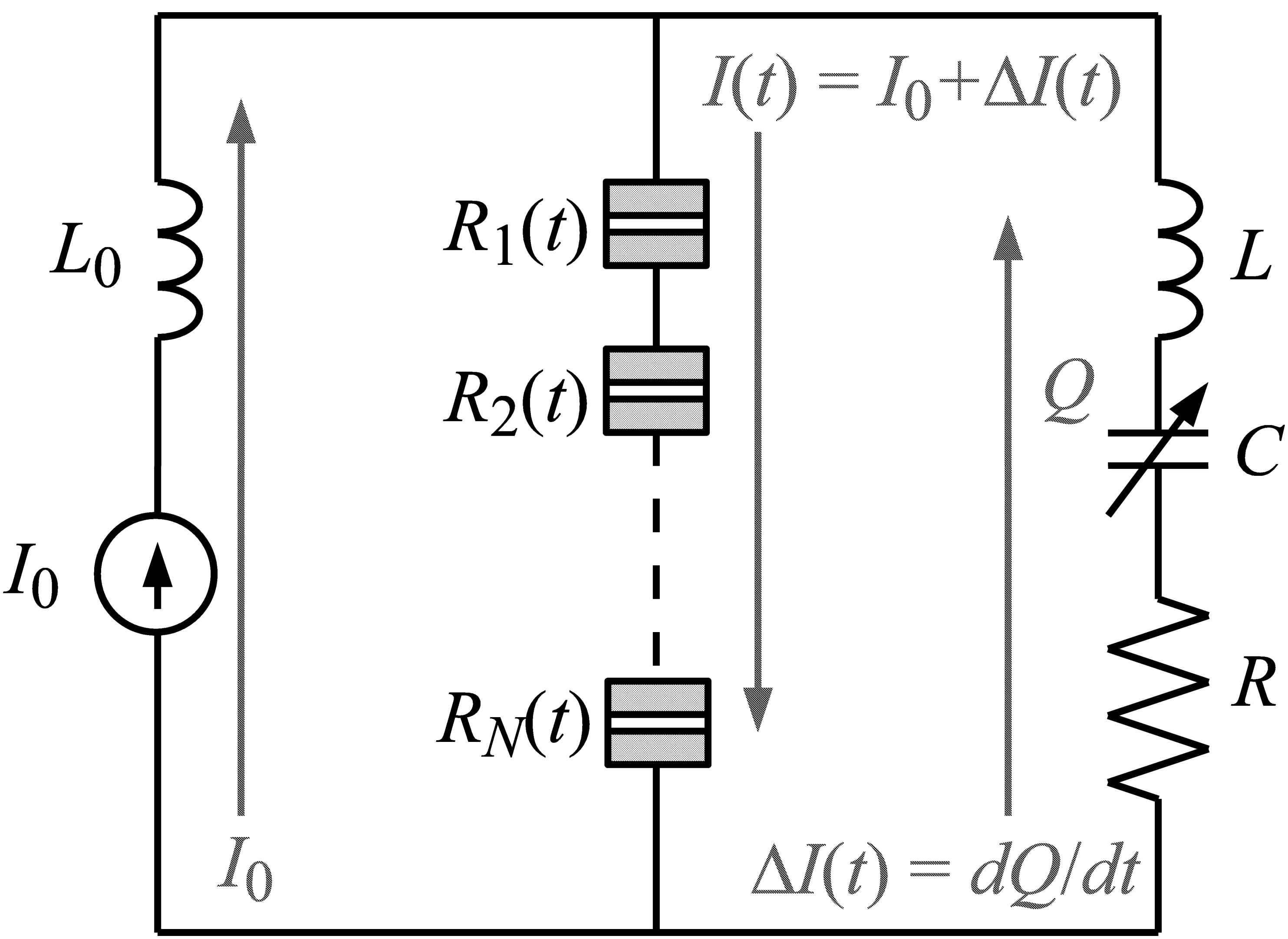
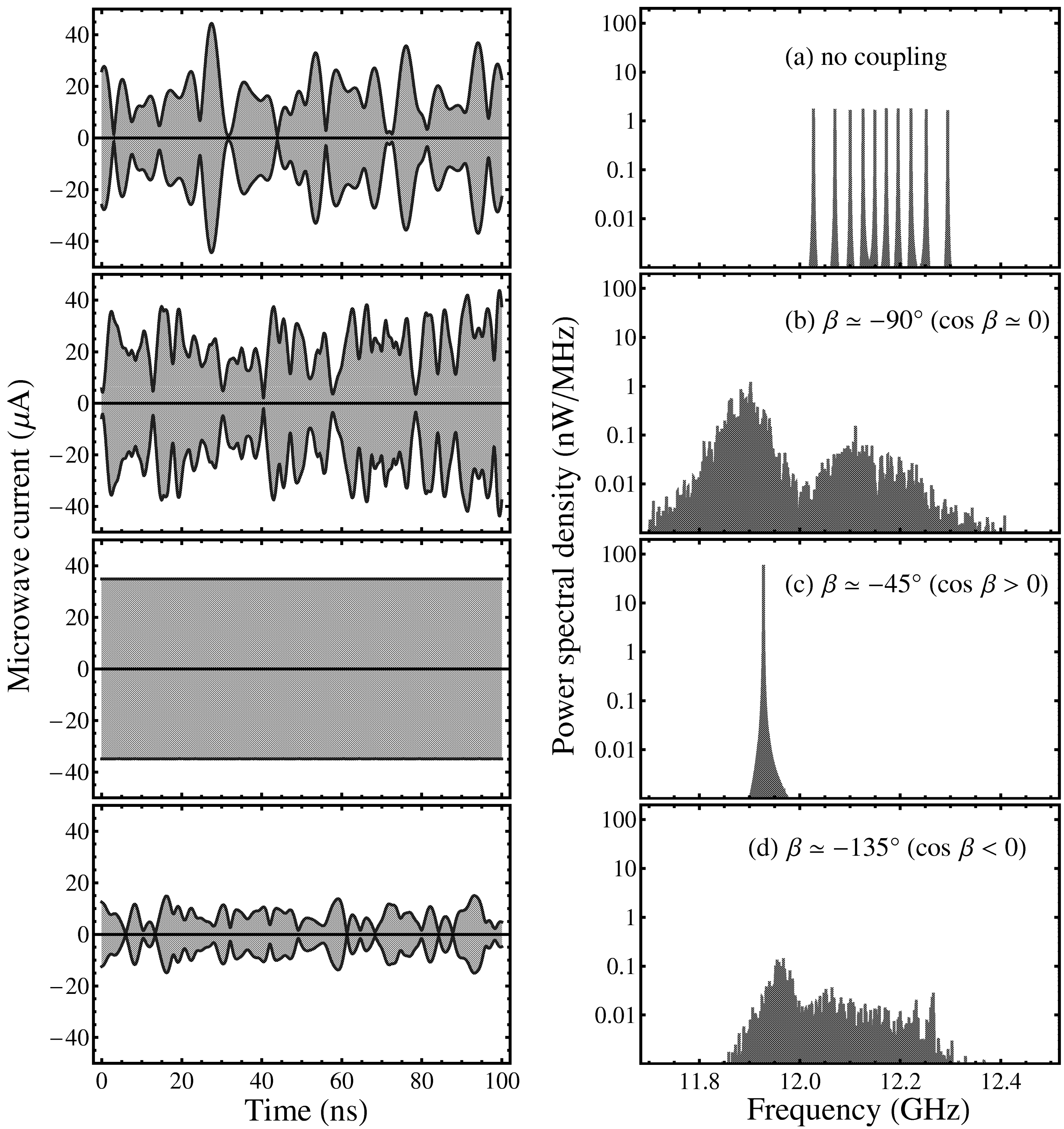
Figure captions

FIG. 1: Scheme of the studied STNO array consisting of N oscillators connected in series and biased by the dc current I_0. The microwave current $\Delta I(t)$, created by the array, flows through the external LCR circuit. By varying the capacitance C of this circuit one can control the phase shift β_c between the microwave current $\Delta I(t)$ and the total microwave voltage $\mathcal{E}(t) = I_0 \sum_i \Delta R_i(t)$ on the array.

FIG. 2: Typical temporal profiles of the microwave current $\Delta I(t)$ (left column) and power spectral density on the active load R (right column) for: (a) – uncoupled STNOs and (b), (c), (d) – coupled STNO array with different resonance frequencies f_c of the RLC circuit: (b) $f_c = f_0$, which corresponds to the zero external phase shift $\beta_c = 0$; (c) $f_c = f_0 + \Delta f_c$, (*positive* external phase shift $\beta_c = 45°$); (d) $f_c = f_0 - \Delta f_c$, (*negative* external phase shift $\beta_c = -45°$). Blue lines in the left panel show the envelopes of the microwave current.



$I(t) = I_0 + \Delta I(t)$
L_0
$R_1(t)$
$R_2(t)$
L
Q
C
I_0
$R_N(t)$
R
I_0
$\Delta I(t) = dQ/dt$



Microwave current (μA)
Time (ns)
Power spectral density (nW/MHz)
Frequency (GHz)
(a) no coupling
(b) β ≃ −90° (cos β ≃ 0)
(c) β ≃ −45° (cos β > 0)
(d) β ≃ −135° (cos β < 0)